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Negative superfluid density: Mesoscopic fluctuations and reverse of the supercurrent through a disordered Josephson junction

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We calculate the Josephson coupling energy $U_J(\phi)$ [related to the supercurrent $I=(2e/\hbar)dU_J/d\phi$] for a disordered normal metal between two superconductors with a phase difference ϕ We demonstrate that the symmetry of the scattering matrix of noninteracting quasiparticles in zero magnetic field implies that $U_J(\phi)$ has a minimum at $\phi=0$ A maximum (that would lead to a π junction or negative superfluid density) is excluded for any realization of the disorder

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The question of whether mesoscopic fluctuation can reverse the supercorrent through a disorderd Josephson junction was posed ten years ago by Spivak and Kivelson,[1] in their search for mechanisms that would lead to a negative local superfluid density in a dirty superconductor A negative instead of a positive superfluid density means that the Josephson coupling energy $U_J(\phi)$ has a maximum instead of a minimum for zero phase difference ϕ of the superconducting order parameter The supercurrent $I(\phi)=(2e/\hbar)dU_J/d\phi$ then has the opposite sign as usual for small ϕ This has a variety of observable consequences, including a ground state with a nonzero supercurrent,[2] Aharonov-Bohm oscillations with period $h/4e$, and negative magnetoresistance [3] A Josephson junction with a negative superfluid density is known as a π junction,[2] because the ground state for large magnetic inductance is close to $\phi=\pi$ instead of as usual at $\phi=0$

The known mechanism for the creation of a π junction[1-4] in equilibrium[5] in a nonmagnetic material[6] requires strong Coulomb repulsion to create a localized spin on a resonant impurity level Spivak and Kivelson asked the question whether purely one-electron conductance fluctuations might be sufficient to produce a locally negative superfluid density near the insulating state A suggestive argument that this might be possible comes from the relative magnitude of the mesoscopic sample-to-sample fluctuations of the supercurrent in a disordered superconductor–normal-metal–superconductor (SNS) junction [7] The ratio $\langle \delta I^2\rangle^{1/2}/\langle I\rangle \simeq e^2/hG$ of the root-mean-squared fluctuations over the mean supercurrent is $\ll 1$ if the conductance G of the normal metal is large compared to the conductance quantum e^2/h The ratio becomes of order unity on approaching the insulating state, suggesting that the supercurrent might have a negative value in some samples

Of course, the root-mean-square amplitude of the supercurrent fluctuations does not distinguish between a positive and negative sign, so that this argument is only suggestive We were motivated to settle this issue because of recent experiments on localization in quasi-one-dimensional superconductors [8] This has renewed the interest in the fundamental question whether mesoscopic fluctuations are sufficient or not to create a negative local superfluid density The answer, as we will show, is that they are not

The two superconductors that form the SNS junction have order parameters $\Delta e^{\imath\phi/2}$ and $\Delta e^{-\imath\phi/2}$ The contacts to the normal metal have N propagating modes at the Fermi energy E_F, so that the elastic scattering by the normal metal at energy $E=E_F+\varepsilon$ is characterized by a $2N\times 2N$ scattering matrix $S(\varepsilon)$ The two properties of S that we use are that it is analytic in the upper half of the complex ε plane and that it is a symmetric matrix, $S(\varepsilon)=S(\varepsilon)^T$, when time-reversal symmetry is preserved

The starting point of our calculation is the relationship derived in Ref 9 between the Josephson coupling energy $U_J(\phi)$ in equilibrium at temperature T and the scattering matrix,

$$U_J=-2k_BT\sum_{n=0}^{\infty} \ln \mathrm{Det}[1-S_A(\imath\omega_n)S_N(\imath\omega_n)] \tag{1}$$

The summation runs over the Matsubara frequencies $\omega_n=(2n+1)\pi k_BT$ The $4N\times 4N$ scattering matrix $S_N(\varepsilon)$ describes the elastic scattering by disorder in the normal metal of noninteracting electron and hole quasiparticles with excitation energy ε,

$$S_N(\varepsilon)=\begin{pmatrix} S(\varepsilon) & 0 \\ 0 & S(-\varepsilon)^* \end{pmatrix} \tag{2}$$

The analytical continuation of S_N from real to imaginary energy ($\varepsilon\to\imath\omega$) follows from $S(\varepsilon)\to S(\imath\omega)$ and $S(-\varepsilon)^*\to S(\imath\omega)^*$ Similarly, the matrix $S_A(\varepsilon)$ describes the Andreev reflection from the superconductors,

$$S_A(\varepsilon)=\alpha(\varepsilon)\begin{pmatrix} 0 & e^{\imath\Lambda\phi/2} \\ e^{-\imath\Lambda\phi/2} & 0 \end{pmatrix}, \tag{3a}$$

$$\alpha(\varepsilon)=\varepsilon/\Delta-\imath\sqrt{1-\varepsilon^2/\Delta^2} \tag{3b}$$

Here Λ is a $2N\times 2N$ diagonal matrix with elements $\Lambda_{jj}=1$ for $1\leqslant j\leqslant N$ and $\Lambda_{jj}=-1$ for $N+1\leqslant j\leqslant 2N$

Eq (1) differs from the usual representation of the Josephson energy as a sum over the discrete spectrum ($\varepsilon<\Delta$) plus an integration over the continuous spectrum ($\varepsilon>\Delta$) The derivation of Eq (1) is based on the analyticity of S_A and S_N in the upper half of the complex ε plane that allows to relate the integration over the real energies to the summation over the Matsubara frequencies Thus each term in the sum (1) is combined from contributions out of the discrete and the continuous spectrum We will now show that each of these combinations is minimal for $\phi=0$, although the contributions from the discrete and continuous spectrum separately are not

Let us abbreviate

$$Z(\omega)=(\sqrt{1+\omega^2/\Delta^2}-\omega/\Delta)e^{-i\Lambda\phi/2}S(i\omega) \tag{4}$$

Using the identity $\ln \mathrm{Det}=\mathrm{Tr}\ln$ in Eq (1) one can calculate the first and second derivative with respect to ϕ of the energy $U_J(\phi)$ The first derivative is given by

$$\frac{dU_J}{d\phi}=2k_BT\sum_{n=0}^{\infty} \mathrm{Im}\,\mathrm{Tr}[h_{11}(\omega_n)-h_{22}(\omega_n)], \tag{5}$$

where the $N\times N$ matrices h_{11} and h_{22} are blocks of the matrix

$$H=Z^*Z(1+Z^*Z)^{-1}=\begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \tag{6}$$

The first derivative is equal to the supercurrent and vanishes at $\phi=0$ as dictated by time-reversal symmetry

For the second derivative we obtain

$$\frac{d^2U_J}{d\phi^2}=4k_BT\sum_{n=0}^{\infty} \mathrm{Re}\,\mathrm{Tr}[f_{12}(\omega_n)f_{21}^{\dagger}(\omega_n)+h_{12}(\omega_n)h_{21}(\omega_n)], \tag{7}$$

$$F=Z(1+Z^{\dagger}Z)^{-1}=\begin{pmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{pmatrix} \tag{8}$$

At $\phi=0$ the symmetry of S implies that $F=F^T$ and $H=H^\dagger$, hence $f_{21}^\dagger=f_{12}^\dagger$, and $h_{21}=h_{12}^\dagger$ Therefore, every term in the sum (7) is positive We conclude that the Josephson energy $U_J(\phi)$ has a minimum at $\phi=0$,

$$\left.\frac{dU_J}{d\phi}\right|_{\phi=0}=0, \quad \left.\frac{d^2U_J}{d\phi^2}\right|_{\phi=0}>0 \tag{9}$$

This concludes the proof that mesoscopic fluctuations cannot invert the stability of the SNS junction at zero phase, excluding a mechanism for the creation of a π junction proposed ten years ago [1] The proof holds for noninteracting quasiparticles in zero magnetic field at arbitrary temperature, for any disorder potential and any dimensionality of the junction As a final remark, we conjecture (and have a proof for $N=1$) that Eq (1) implies $dU_J/d\phi>0$ in the entire interval $0<\phi<\pi$ in the presence of time-reversal symmetry

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